A New Approach to Design of a Dynamic Output Feedback Stabilizing Control Law for LTI Systems

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We present a new state-space approach to construct a dynamic output feedback controller which stabilizes a class of linear time invariant systems. All the states of the given system are not measurable and only the output is used to design the stabilizing control law. In the design scheme, however, we first assume that the given system can be stabilized by a feedback law composed of the output and its derivatives of a certain order. Beginning with this assumption, we systematically construct a dynamic system which removes the need of the derivatives. The main advantage of the proposed controller is regarding the controller order, which may be smaller than that of conventional output feedback controller. Using a simple numerical example, it is shown that the order of the proposed controller is indeed smaller than that of reduced-order observer based output feedback controller.

Key Words : Linear System, Stability, State Space, Output Derivatives, Dynamic Output Feedback, Recursive Design

1. Introduction

In this paper, we consider the stabilization problem of a system represented by

$$\dot{x}=Ax+Bu \qquad (1)$$
$$y=Cx$$

where x is the state in R^n, u is the input in R^m, y is the measurable output in R^p.

We suppose that the system (1) is not able to be stabilized by any static output feedback (Syrmos et al., 1997). When the measurable states are not sufficient to design a stabilizing control law, a dynamic output feedback scheme with an additional dynamic system e.g. state observer is designed so that the augmented closed loop system is stable (Kailath, 1980; Chen, 1984; Shim et al., 2003; Jo and Son, 2004). When dynamic output feedback controllers are concerned, most researches are concentrated on the arbitrary pole-placement rather than stabilization (see e.g. Rosenthal and Wang, 1996; Scherer et al., 1997) and references therein). However, if we restrict

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our interests to the stabilization problem, like the static output feedback control problem, the order of the controllers can be reduced (Son et al, 2000, Son et al, 2002b)

While the measurable states are not sufficient to design a stabilizing static control law, this paper assumes that a static state feedback exists for stabilization if the output and its derivatives of a certain order are available to be used Then, we present a new way to replace the required derivatives by adding some dynamics in the feedback This is, in fact, inspired by (Son et al, 2002a), where a passivity-based dynamic output feedback control has been proposed for inherently non-passive LTI systems by virtue of paralleling a feedforward compensator In (Son et al, 2002a), it has also been observed that, when a system is stabilized by a proportional-derivative control, the derivative term can be replaced* with a compensator which has the same dimension as the system's input The idea of replacing the derivative term is further exploited in this paper up to any order

The only assumption in this paper is the following

Assumption 1 Let us define

$$G_k = [K_0\ K_1\ \cdots\ K_k] \text{ and } H_k = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^k \end{bmatrix}$$

For the system (1), there exists an integer r ($1 \leq r$) such that

$$A_r = A + BG_rH_r \text{ is Hurwitz}$$

Remark 1 It is presumed in this assumption that $r \geq 1$ because, when Assumption 1 holds with $r=0$, the system (1) can be trivially stabilized by a static output feedback without using additional dynamics On the other hand, if the system (1) is stabilizable and observable, then Assumption 1 trivially holds with $r=n-1$ Indeed, in this case, H_r is left-invertible due to observability, and thus, there always exists G_r with which Assumption 1 holds

In the next section, a dynamic output feedback controller is presented for system (1) under Assumption 1, followed by a recursive algorithm to design the gains of the proposed controller in a systematic manner Section 3 illustrates a design example with a simulation result Conclusions are found in Section 4

Notations I_n is an identity matrix and $0_{m\times p} \in \mathrm{R}^{m\times p}$ is a zero matrix

2. Main Results

For the system (1) satisfying Assumption 1, we propose a dynamic output feedback controller of order pr, which has the form of

$$\dot{\lambda} = \Psi_a y + \Psi_b \lambda,\ \lambda \in \mathrm{R}^{pr},\ u = \Phi_a y + \Phi_b \lambda \qquad (2)$$

The output feedback stabilization problem is solved if we find $\Psi = [\Psi_a\ \Psi_b]$ and $\Phi = [\Phi_a\ \Phi_b]$ such that the following closed-loop system

$$\begin{aligned} \dot{x} &= Ax + B\Phi_a Cx + B\Phi_b \lambda \\ \dot{\lambda} &= \Psi_a Cx + \Psi_b \lambda \end{aligned} \qquad (3)$$

is exponentially stable

In the subsequent part of the paper, we propose a new way to design the matrices Ψ and Φ Therefore, the main contribution of the paper is summarized as follows

Theorem 1 For the system (1) satisfying Assumption 1, there exists a dynamic output feedback stabilizing controller (2) with additional λ-dynamics of order ($p \times r$)

The idea of constructing the controller (2) is to assume, temporarily in the beginning, that $H_r x$ is available for measurement This makes the output feedback stabilization problem be solved by the static gain found in Assumption 1 Next, we change the temporary assumption such that $H_{r-1}x$ is available for measurement but $H_r x$ is not (This implies that $CA^i x$, $i=0, \cdots, r-1$, is

* The design of a dynamic system for replacing the velocity measurement has been studied by several authors (Kaufman et al, 1998, Kelly et al 1994, Fujisaki et al, 2001, Wong et al, 2001)

measurable but $CA^r x$ is not.) Then, the control law designed at the previous step, where we assumed that $H_r x$ is measurable, is not implementable because it depends on the signal $CA^r x$. Hence, we separate the term $CA^r x$ from the control law and design additional dynamics with which the use of $CA^r x$ is eliminated. In the next step, we proceed by assuming that $H_{r-2}x$ is measurable but $CA^r x$ is not. This recursion goes to the end if we get a dynamic controller that requires only the true measurement of $H_0 x = Cx$ but not others

The recursion begins by the following initial step

2.1 Initial step

When the $H_r x$ is measurable, we easily obtain the following stable closed loop system S_r with the gain G_r from Assumption 1

$$S_r : \begin{cases} u = G_r H_r x \\ \quad = G_{r-1}H_{r-1}x + K_r(CA^r x) \\ x = A_r x = (A + BG_r H_r)x \\ \quad = Ax + BG_{r-1}H_{r-1}x + BK_r(CA^r x) \end{cases} \tag{4}$$

Now, we assume that $H_{r-1}x$ is available for measurement but $CA^r x$ is not Then, by introducing v, we decompose the system S_r into the term including $CA^r x$ and the rest (as follows):

$$u = G_{r-1}H_{r-1}x + K_r v \tag{5a}$$

$$\dot{x} = Ax + BG_{r-1}H_{r-1}x + BK_r v \tag{5b}$$

If the following dynamic system is appended to (5b)

$$\begin{aligned} \lambda = & -CA^{r-1}BG_{r-1}H_{r-1}x \\ & -(I_p + CA^{r-1}BK_r)v \end{aligned} \tag{6a}$$

$$\bar{y} = CA^{r-1}x + \lambda \tag{6b}$$

then the augmented system (5b)-(6a) is stabilized by $v = D_r\bar{y}$ where D_r is chosen so that the following matrix is Hurwitz

$$\begin{bmatrix} A_r & -A_r BK_r \\ CA^r & -CA^r BK_r - D_r \end{bmatrix}$$

Proof of Initial Step

First of all, note that

$$\begin{aligned} \frac{d}{dt}\bar{y} &= CA^{r-1}\dot{x} + \lambda \\ &= CA^{r-1}(Ax + BG_{r-1}H_{r-1}x + BK_r v) \\ &\quad -(CA^{r-1}BG_{r-1}H_{r-1}x + CA^{r-1}BK_r v + v) \\ &= CA^r x - v \end{aligned}$$

We now define

$$\xi \;\; = x + BK_r\bar{y} \tag{8}$$

and change coordinates $[x^T \; \lambda^T]^T$ into $[\xi^T \; \bar{y}^T]^T$ Then

$$\begin{aligned} \xi &= A_r\xi - A_r BK_r\bar{y} \\ \bar{y} &= CA^r\xi - CA^r BK_r\bar{y} - v \end{aligned} \tag{9}$$

Since the matrix A_r is Hurwitz, the system (9) can be stabilized by $v = D_r\bar{y}$ with an appropriate gain D_r making the matrix (7) Hurwitz For example, $D_r = d_r I_p$ with sufficiently large $d_r > 0$ always performs this task. ◇

Consequently, we obtain the closed loop system S_{r-1} as follows

$$\begin{cases} u = G_{r-1}H_{r-1}x + K_r D_r(CA^{r-1}x + \lambda) \\ \quad = (G_{r-1} + [0_{m\times p(r-1)} \; K_r D_r])H_{r-1}x + K_r D_r\lambda \\ \dot{x} = Ax + BG_{r-1}H_{r-1}x + BK_r D_r(CA^{r-1}x + \lambda) \\ \quad = Ax + B(G_{r-1} + [0_{m\times p(r-1)} \; K_r D_r])H_{r-1}x + BK_r D_r\lambda \\ \lambda = -(I + CA^{r-1}BK_r)D_r(CA^{r-1}x + \lambda) - CA^{r-1}BG_{r-1}H_{r-1}x \\ \quad = -(CA^{r-1}BG_{r-1} + [0_{p\times p(r-1)} \; M_\lambda])H_{r-1}x - M_\lambda\lambda \end{cases} \tag{10}$$

where $M_\lambda = (I_p + CA^{r-}BK_r)D_r$ The above system (10) is stable because its system matrix is similar to the matrix (7)

2.2 Recursive design of output feedback controller

We assume that, with some integer k between 1 and r, it holds that $H_k x$ is measurable and the following output feedback controller of order $p(r-k)$ stabilizes system (1) exponentially.

$$\begin{aligned} \lambda &= \Psi_{k,a}H_k x + \Psi_{k,b}\lambda \\ u &= \Phi_{k,a}H_k x + \Phi_{k,b}\lambda \end{aligned} \tag{11}$$

where $\Phi_{k,a}$, $\Phi_{k,b}$, $\Psi_{k,a}$ and $\Psi_{k,b}$ are matrices of appropriate dimension In other words, the closed-loop system

$$S_k=\begin{cases}\dot{x}=Ax+B\Phi_{k,a}H_kx+B\Phi_{k,b}\lambda\\ \dot{\lambda}=\Psi_{k,a}H_kx+\Psi_{k,b}\lambda\end{cases} \tag{12}$$

is exponentially stable, which can be concisely represented by

$$\dot{z}=A_kz \tag{13}$$

where $z=[x^T\ \lambda^T]^T$ and the Hurwitz matrix A_k is defined as

$$A_k=\begin{bmatrix}A+B\Phi_{k,a}H_k & B\Phi_{k,b}\\ \Psi_{k,a}H_k & \Psi_{k,b}\end{bmatrix} \tag{14}$$

Now we postulate a new assumption that $H_{k-1}x$ is measurable but CA^kx is not, so that the controller (11) cannot be implemented Thus, we separate the term CA^kx from the controller equation (11) and replace it by a new signal v to be designed as follows ·

$$\begin{aligned}\dot{\lambda}&=\Psi_{k,a1}H_{k-2}x+\Psi_{k,b}\lambda+\Psi_{k,a2}CA^kx\\ &=\Psi_{k,a1}H_{k-2}x+\Psi_{k,b}\lambda+\Psi_{k,a2}v\\ u&=\Phi_{k,a1}H_{k-2}x+\Phi_{k,b}\lambda+\Phi_{k,a2}CA^kx\\ &=\Phi_{k,a1}H_{k-2}x+\Phi_{k,b}\lambda+\Phi_{k,a2}v\end{aligned} \tag{15}$$

where $\Psi_{k,a}=[\Psi_{k,a1}\ \Psi_{k,a2}]$ and $\Phi_{k,a}=[\Phi_{k,a1}\ \Phi_{k,a2}]$ Then, the closed-loop system is rewritten by

$$\begin{aligned}\dot{x}&=Ax+B\Phi_{k,a1}H_{k-1}x+B\Phi_{k,b}\lambda+B\Phi_{k,a2}v\\ \dot{\lambda}&=\Psi_{k,a1}H_{k-1}x+\Psi_{k,b}\lambda+\Psi_{k,a2}v\end{aligned} \tag{16}$$

or

$$\dot{z}=Fz+Lv \tag{17}$$

where

$$F=\begin{bmatrix}A+B\Phi_{k,a1}H_{k-1} & B\Phi_{k,b}\\ \Psi_{k,a1}H_{k-1} & \Psi_{b,b}\end{bmatrix},\ L=\begin{bmatrix}B\Phi_{k,a2}\\ \Psi_{k,a2}\end{bmatrix}$$

which is equivalent to (12) (or to (13)) if $v=CA^kx$ Note that $A_k=F+L[CA^k\ 0_{p\times p(r-k)}]$

The following theorem provides a key to the recursion in the sense that it shows how to replace CA^kx term by an additional dynamics

Theorem 1 Suppose that system (16) (or, (17)) is exponentially stable if $v=CA^kx$, that is, the matrix A_k is Hurwitz. If the following dynamic system is appended to (16) (or, (17))

$$\begin{aligned}\dot{\eta}=&-CA^{k-1}B\Phi_{k,a1}H_{k-1}x-CA^{k-1}B\Phi_{k,b}\lambda\\ &-(I_p+CA^{k-1}B\Phi_{k,a2})v,\ \eta\in\mathbb{R}^p\end{aligned} \tag{18a}$$

$$\bar{y}=CA^{k-1}x+\eta \tag{18b}$$

then the augmented system (16), (18) (or, (17), (18)) is exponentially stabilized by

$$v=D_k\bar{y} \tag{19}$$

where the matrix D_k is chosen such that

$$\begin{bmatrix}A_k & -A_kL\\ [CA^k0_{p\times p(r-k)}] & -CA^kB\Phi_{k,a2}-D_k\end{bmatrix} \tag{20}$$

is Hurwitz

Remark 4 Note that the matrix (20) always can be made Hurwitz by appropriate matrix D_k, which can be found by LMI tool or by choosing sufficiently large constant $d_k>0$ and letting $D_k=d_kI_p$

Proof With the control law (18) and (19), the closed-loop system is given by (17) and (18a) with (19) In order to analyze its stability, the closed-loop system is represented in the $(z,\bar{y})$-coordinates instead of (z,η) That is, the closed-loop system is now given by (17) and

$$\begin{aligned}\frac{d}{dt}\bar{y}=&CA^{k-1}(Ax+B\Phi_{k,a1}H_{k-1}x+B\Phi_{k,b}\lambda+B\Phi_{k,a2}v)\\ &-CA^{k-1}B\Phi_{k,a1}H_{k-1}x-CA^{k-1}B\Phi_{k,b}\lambda\\ &-(I_p+CA^{k-1}B\Phi_{k,a2})v\\ =&CA^kx-v\end{aligned}$$

Now we change the coordinates $(z,\bar{y})$ into $(\xi,\bar{y})$ once again with a new variable $\xi=z-L\bar{y}$ That is,

$$\begin{aligned}\dot{\xi}&=(Fz+Lv)+L(CA^kx-v)=A_kz\\ &=A_k\xi-A_kL\bar{y}\\ \frac{d}{dt}\bar{y}&=[CA^k\ 0_{p\times p(r-k)}]z-v\\ &=[CA^k\ 0_{p\times p(r-k)}]\xi-[CA^k\ 0_{p\times p(r-k)}]L\bar{y}-v\\ &=[CA^k\ 0_{p\times p(r-k)}]\xi-CA^kB\Phi_{k,a2}\bar{y}-v\\ v&=D_k\bar{y}\end{aligned}$$

Therefore, it is seen that if D_k is chosen such that *the matrix (20) is Hurwitz, the above* closed-loop system is exponentially stable ◇

Remark 5. As a result of Theorem 3, it follows that the overall closed-loop system, which is obtained from (16), (18) and (19), is exponentially stable The single equation (21) is the closed-loop system, whose system matrix will become the matrix A_{k-1} in the next iteration step

$$\begin{bmatrix}\dot{x}\\ \dot{\lambda}\\ \dot{\eta}\end{bmatrix}=\begin{bmatrix}A+B\Phi_{k,a1}H_{k-1}+B\Phi_{k,a2}D_kCA^{k-1} & B\Phi_{k,b} & B\Phi_{k,a2}D_k\\ \Psi_{k,a1}H_{k-1}+\Psi_{k,a2}D_kCA^{k-1} & \Psi_{k,b} & \Psi_{k,a2}D_k\\ -CA^{k-1}B\Phi_{k,a1}H_{k-1}-(I+CA^{k-1}B\Phi_{k,a2})D_kCA^{k-1} & -CA^{k-1}B\Phi_{k,b} & -(I+CA^{k-1}B\Phi_{k,a2})D_k\end{bmatrix}\begin{bmatrix}x\\ \lambda\\ \eta\end{bmatrix} \quad (21)$$

The recursion procedure is now quite obvious Since $k=r$ at the initial step, $\Psi_{r,a}$, $\Psi_{r,b}$ and $\Phi_{r,b}$ are null matrices (i e , empty) and the controller (18) becomes just a static feedback $u=G_rH_rx$ (i e , $\Phi_{r,a}=G_r$) from Assumption 1 Therefore, we have the Hurwitz matrix $A_r=A+BG_rH_r$ By Theorem 3, unmeasurable term CA^rx is replaced by the dynamic controller (18) and (19) Now, we regard the state η of (18) as the state λ of (11) (i.e. (6) and (10)) for the next iteration (The next step begins with the equation (11)) In particular, from (10) it is obtained that

$$\Psi_{r-1,a}=-(CA^{r-1}B\Phi_{r,a1}+[0_{p\times p(r-1)}\ (I_p+CA^{r-1}B\Phi_{r,a2})D_r])$$
$$\Psi_{r-1,b}=-(I_p+CA^{r-1}B\Phi_{r,a2})D_r$$
$$\Phi_{r-1,a}=\Phi_{r,a1}+[0_{m\times p(r-1)}\ \Phi_{r,a2}D_r]$$
$$\Phi_{r-1,b}=\Phi_{r,a2}D_r$$

where $\Phi_{r,a1}=G_{r-1}$ and $\Phi_{r,a2}$. Likewise, the iteration proceeds until we have a controller of (11) with $k=0$ Therefore, we obtain the gains of (2) as follows

$$\Psi_a=\Psi_{0,a},\ \Psi_b=\Psi_{0,b},\ \Phi_a=\Phi_{0,a},\ \Phi_b=\Phi_{0,b}$$

For convenience, we include a formula for the iteration

$$\Psi_{k-1,a}=\begin{bmatrix}\Phi_{k,a1}+[0_{p(r-k)\times p(k-1)}\ \Psi_{k,a2}D_k]\\ -CA^{k-1}B\Phi_{k,a1}-[0_{p\times p(k-1)}\ (I_p+CA^{k-1}B\Phi_{k,a2})D_k]\end{bmatrix} \quad (22a)$$

$$\Psi_{k-1,b}=\begin{bmatrix}\Psi_{k,b} & \Psi_{k,a2}D_k\\ -CA^{k-1}B\Phi_{k,b} & -(I_p+CA^{k-1}B\Phi_{k,a2})D_k\end{bmatrix} \quad (22b)$$

$$\Phi_{k-1,a}=\Phi_{k,a1}+[0_{m\times p(k-1)}\ \Phi_{k,a2}D_k] \quad (22c)$$

$$\Phi_{k-1,a}=[\Phi_{k,b}\ \Phi_{k,a2}D_k] \quad (22d)$$

3. An Illustrative Example

We illustrate the proposed design method with a simple numerical example.

$$\dot{x}=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&-6\end{bmatrix}x+\begin{bmatrix}0\\0\\0\\1\end{bmatrix}u \quad (23)$$
$$y=[1\ 0\ 0\ 0]x$$

For (23) we can see that the order of the proposed controller $(p\times r)$ is less than that of the reduced order observer based output feedback controller $(n-1)$

The system (23) satisfies Assumption 1 with $r=2$ Hence, the order of the proposed controller is two, while the reduced order observer based controller has order three In fact, with the following control law

$$u=G_2H_2x=[-6\ -12\ -13]H_2x \quad (24)$$

the eigenvalues of the matrix $A_2=A+BG_2H_2$ are given by $\{-2\,30\pm j0\,625,\ -0\,70\pm j0\,625\}$ Hence, the closed loop system (23)-(24) is stable and we obtain $G_1=[-6\ -12]$ and $K_2=13$ for the iteration

Now, in order to replace the CA^2x-term in H_2x as the initial step, we consider the matrix of (7) for the system (23) Indeed, with $D_2=20$, the matrix (7) is given by

$$\begin{bmatrix}0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&13\\-5&-12&-13&-6&-78\\0&0&1&0&-20\end{bmatrix} \quad (25)$$

which is Hurwitz

However, since the CAx-term in H_1x is neither measurable, we proceed one step further by Theorem 3 From the previous step and the equation (22), the parameters of (11) can be regarded as

$$\Psi_{1,a}=[0\ -20],\ \Psi_{1,b}=-20$$
$$\Phi_{1,a}=[-6\ -272],\ \Phi_{1,b}=-260 \quad (26)$$

With these parameters the matrix A_1 in (14) is given by

$$A_1=\begin{bmatrix}0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\-5&-272&0&-6&-260\\0&-20&0&0&20\end{bmatrix} \quad (27)$$

Hence, the gain D_1 is chosen such that the matrix in (20) is Hurwitz, which is achieved by $D_1=30$

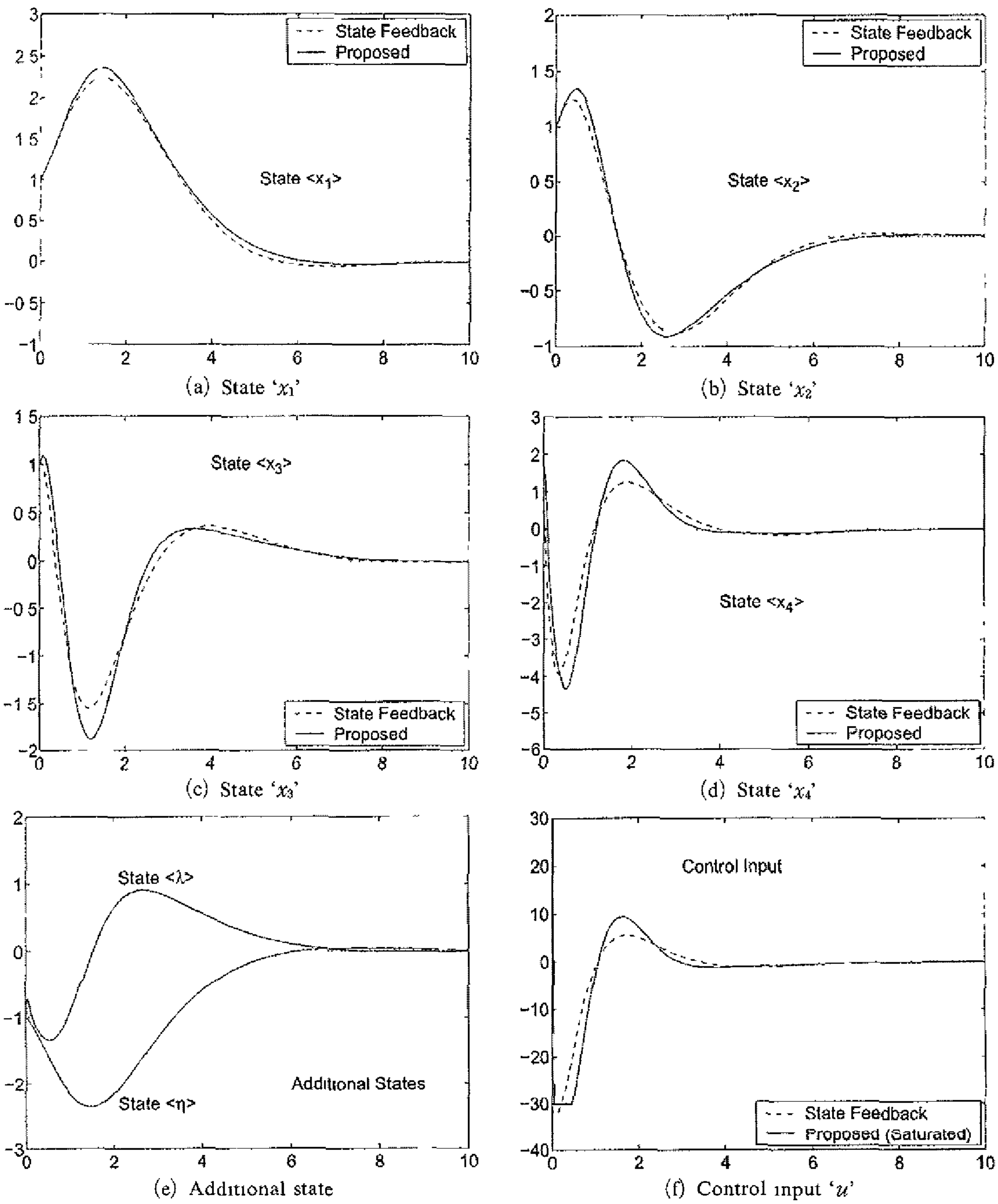
Therefore, with the following additional dynamics

$$\begin{cases} \dot{\lambda} = -600y - 20\lambda - 600\eta \\ \eta = -30y - 30\eta \end{cases} \tag{28}$$

the stabilizing control law for (23) is obtained by

$$u = -8166y - 260\lambda - 8160\eta \tag{29}$$

Figure 1 shows the simulation result (solid curve) of the proposed controller. In the simulation, we added a saturation (whose level is



(a) State 'x_1' (b) State 'x_2'

(c) State 'x_3' (d) State 'x_4'

(e) Additional state (f) Control input 'u'

Fig. 1 Simulation Results (proposed solid)

30) to (29) In the figure, we also compared the plots with the results (dotted curve) obtained from the state feedback control (24) We can see that the additional dynamics successfully replace the derivative terms CAx and CA^2x in (24) All the initial conditions of the systems are set to while all the initial states of the additional dynamics are set to -1

4. Conclusion

In this paper, we have presented a new recursive algorithm to design a dynamic output feedback control law which stabilizes linear time-invariant systems If a given plant can be stabilized by a static feedback of the output and its derivatives, the proposed method systematically constructs a dynamic system which successfully replaces the output derivative terms of any order without any additional conditions A numerical example with a simulation result has been presented to illustrate the design method From the proposed recursion algorithm, it is not difficult to develop an automated design package on a PC

Acknowledgment

The first and the last authors would like to thank Korea Ministry of Science and Technology and Korea Science and Engineering Foundation for their support through ERC program This work was also supported by Korea Science and Engineering Foundation Grant (KOSEF-R05-2003-000-10624-0) and (KOSEF-R-01-2003-000-10014-0)

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